Lesson 3-6: Slopes of Parallel and Perpendicular Lines

Parallel lines

How do parallel lines relate with each other? Well we know from the definition of parallel lines that they are coplanar and never intersect. How can we express that algebraically? In other words, can we use analytic geometry to look at the equations of two lines and determine if they are parallel?

Slopes of parallel lines

Last lesson we recalled that slope is an indication of the tilt (and direction) of a line. If two lines are parallel, what would you think you could say about their tilt and direction? They are the same (otherwise the lines would eventually intersect). This leads to the conclusion that their slopes are the same. Here are three statements you can accept as fact:

1. Two non-vertical lines are parallel, ***iff*** their slopes are equal.
2. Any two vertical lines are parallel.
3. Any two horizontal lines are parallel.

Let's put this into action...

Example – Pg 161, #2

Are lines 1 and 2 parallel? Explain using slope:

Line 1: (1, 3) & (-5, 1) $\rightarrow$ $m = \dfrac{3-1}{1-(-5)} = \dfrac{2}{6} = \dfrac{1}{3}$

Line 2: (4, 2) & (-4, -2) $\rightarrow$ $m = \dfrac{2-(-2)}{4-(-4)} = \dfrac{4}{8} = \dfrac{1}{2}$

Not parallel, diff slopes

Example – Pg 161, #10

Are the lines parallel? Explain:

$3x + 4y = 12 \rightarrow$ $y = -\dfrac{3}{4} + 3, m = -\dfrac{3}{4}$

$6x + 2y = 6 \rightarrow y = -3x + 3$, $m = -3$

Not parallel, diff slopes

Example – Pg 161, #14

Write an equation for the line parallel to line *AB* that contains point *C*:

Line AB: $-x + 2y = 4$, $C(-2, 4)$, $m = \dfrac{1}{2}$

$y - 4 = \dfrac{1}{2}(x + 2)$

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Perpendicular lines

OK, we have parallel lines down, what about perpendicular lines? Again, we know they are two lines that form a right angle at the point of intersection. What can we say about their slopes?

On a piece of graph paper, graph two perpendicular lines. Do not use a vertical and horizontal line; remember from yesterday that the slope of a vertical line is undefined? It is pretty hard to compare a known number against an undefined number isn't it? Once you have the two lines, determine their slopes. What do you notice? Try to state what you find as a biconditional.

1. Two lines are perpendicular ***iff*** the product of their slopes is -1.
2. Any horizontal and vertical line are perpendicular.

Got it? Try it out on the following:

Example – Pg 162, #18

Are the two lines perpendicular? Explain using slope:

Line 1: (6, 3) & (1, -1) $\rightarrow m_1 = \frac{3-(-1)}{6-1} = \frac{4}{5}$

Line 2: (-4, 4) & (1, -1) $\rightarrow m_2 = \frac{4-(-1)}{-4-1} = \frac{5}{-5} = -1$

$m_1 * m_2 = -\frac{4}{5} \neq -1 \rightarrow$ No they are not perpendicular

Example – Pg 162, #22

Write an equation for the line perpendicular to line *MN* containing point *P*:

Line MN: $y + 2x = -8$; $P(4, 4) \rightarrow y = -2x - 8 \rightarrow m = -2$

Perpendicular line: $m = \frac{1}{2} \rightarrow y - 4 = \frac{1}{2}(x - 4)$

Example – Pg 162, #24

Find an equation for the line for the new road:

$m = \frac{2}{3}$

Perpendicular line: $m = -\frac{3}{2}$, $b = 0 \rightarrow y = -\frac{3}{2}x$

Assign homework

p. 161 #1-33 odd, 37, 41-45, 47, 48